

# A Game-Theoretic Approach to Energy-Efficient Modulation in CDMA Networks with Delay Constraints

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**Abstract**—A game-theoretic framework is used to study the effect of constellation size on the energy efficiency of wireless networks for M-QAM modulation. A non-cooperative game is proposed in which each user seeks to choose its transmit power (and possibly transmit symbol rate) as well as the constellation size in order to maximize its own utility while satisfying its delay quality-of-service (QoS) constraint. The utility function used here measures the number of reliable bits transmitted per joule of energy consumed, and is particularly suitable for energy-constrained networks. The best-response strategies and Nash equilibrium solution for the proposed game are derived. It is shown that in order to maximize its utility (in bits per joule), a user must choose the lowest constellation size that can accommodate the user's delay constraint. Using this framework, the tradeoffs among energy efficiency, delay, throughput and constellation size are also studied and quantified. The effect of trellis-coded modulation on energy efficiency is also discussed.

**Index Terms**—Energy efficiency, M-QAM modulation, game theory, utility function, delay, QoS, cross-layer design.

## I. INTRODUCTION

Wireless networks are expected to support a variety of applications with diverse quality-of-service (QoS) requirements. Because of the scarcity of network resources (i.e., energy and bandwidth), radio resource management is crucial to the performance of wireless networks. Adaptive modulation has been shown to be an effective method for improving the spectral efficiency in wireless networks (see for example [1]–[4]). However, the focus of many of the studies to date has been on maximizing the throughput of the network, and the impact of the modulation order on energy efficiency has not been studied to the same extent. In [5], the authors have used a convex-optimization approach to study modulation optimization for an energy-constrained time-division-multiple-access (TDMA) network.

Game-theoretic approaches to power control have recently attracted considerable attention (see, for example, [6] and the references therein). In this work, we study the effects

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of modulation on energy efficiency of code-division-multiple-access (CDMA) networks using a *competitive* multiuser setting. Focusing on M-QAM modulation, we propose a non-cooperative game in which each user chooses its strategy, which includes the choice of the transmit power, transmit symbol rate and constellation size, in order to maximize its own utility while satisfying its QoS constraints. The utility function used here measures the number of reliable bits transmitted per joule of energy consumed, and is particularly suitable for energy-constrained networks. Using our non-cooperative game-theoretic framework, we quantify the tradeoffs among energy efficiency, delay, throughput and modulation order. While game-theoretic approaches to resource allocation with delay QoS constraints have previously been studied in [7] and [8], this is the first work that takes into account the effect of modulation.

The remainder of this paper is organized as follows. The system model and definition of the utility function are given in Section II. We first discuss our proposed power control game without any delay constraints in Section III and derive the corresponding Nash equilibrium solution. The delay-constrained power control game is presented in Section IV and the corresponding best-response strategies and Nash equilibrium solution are derived. The analysis is extended to coded systems in Section V. Numerical results and conclusions are given in Sections VI and VII, respectively.

## II. SYSTEM MODEL

We consider a wireless network in which the users' terminals are transmitting to a common concentration point (e.g., a base station or an access point). We define the utility function of a user as the ratio of its goodput to its transmit power, i.e.,

$$u_k = \frac{T_k}{p_k}. \quad (1)$$

Goodput is the net number of information bits that are transmitted without error per unit time and is expressed as

$$T_k = R_k f(\gamma_k) \quad (2)$$

where  $R_k$  is the transmission rate,  $\gamma_k$  is the output signal-to-interference-plus-noise ratio (SIR) for user  $k$ , and  $f(\gamma_k)$  is the “efficiency function” which represents the packet success rate (PSR). We require that  $f(0) = 0$  to ensure that  $u_k = 0$  when  $p_k = 0$ . In general, the efficiency function depends on the modulation, coding and packet size. Based on (1) and (2), the utility function for user  $k$  can be written as

$$u_k = R_k \frac{f(\gamma_k)}{p_k}. \quad (3)$$

This utility function, which has units of *bits/joule*, measures the number of reliable bits that are transmitted per joule of energy consumed, and is particularly suitable for energy-constrained networks.

Focusing on M-QAM modulation, in this work we study non-cooperative games in which the actions open to each user are the choice of transmit power (and possibly transmit symbol rate) as well as the choice of *constellation size*. For the M-QAM modulation, the number of bits transmitted by each symbol is given by

$$b = \log_2 M. \quad ^1$$

We focus on *square* M-QAM modulation, i.e.,  $b \in \{2, 4, 6, \dots\}$ , since there are exact expressions for the symbol error probability of square M-QAM modulation (see [9]). We can easily generalize our analysis to include odd values of  $b$  by using an approximate expression for the symbol error probability. Let us for now focus on a specific user and drop the subscript  $k$ . Assuming a packet size of  $L$  bits, the packet success rate for square M-QAM modulation is given by

$$P_{\text{success}}(b, \gamma) = \left(1 - \alpha_b Q(\sqrt{\beta_b \gamma})\right)^{\frac{2L}{b}} \quad (4)$$

where

$$\alpha_b = 2 \left(1 - 2^{-b/2}\right)$$

and

$$\beta_b = \frac{3}{2^b - 1}.$$

Here,  $\gamma$  represents the symbol SIR and  $Q(\cdot)$  is the complementary cumulative distribution function of the standard Gaussian random variable. Note that at  $\gamma = 0$ , we have  $P_{\text{success}} = 2^{-L} \neq 0$ . Since we require the efficiency function to be zero at zero transmit power, we define

$$f_b(\gamma) = \left(1 - \alpha_b Q(\sqrt{\beta_b \gamma})\right)^{\frac{2L}{b}} - 2^{-L}. \quad (5)$$

Note that  $2^{-L} \simeq 0$  when  $L$  is large (e.g.,  $L = 100$ ).

### III. POWER CONTROL GAME WITH M-QAM MODULATION

We consider a direct-sequence CDMA (DS-CDMA) network with  $K$  users and express the transmission rate of user  $k$  as

$$R_k = b_k R_{s,k} \quad (6)$$

where  $b_k$  is the number of information bits per symbol and  $R_{s,k}$  is the symbol rate. Let us for now assume that users have no delay constraints. We propose a power control game

<sup>1</sup>Since there is a one-to-one mapping between  $M$  and  $b$ , we sometimes refer to  $b$  as the constellation size.

TABLE I

$b$	$\alpha_b$	$\beta_b$	$\gamma_b^*(\text{dB})$	$f_b(\gamma_b^*)$	$b/\gamma_b^*(\text{dB})$	$b f_b(\gamma_b^*)/\gamma_b^*$
2	1	1	9.1	0.801	-6.1	0.1978
4	1.5	0.2	15.7	0.785	-9.7	0.0846
6	1.75	0.0476	21.6	0.771	-13.8	0.0322
8	1.875	0.0118	27.3	0.757	-18.3	0.0112
10	1.9375	0.0029	33.0	0.743	-23.0	0.0037

in which each user seeks to choose its constellation size and transmit power in order to maximize its own utility, i.e.,

$$\max_{b_k, p_k} R_k \frac{f(\gamma_k)}{p_k} \quad \text{for } k = 1, \dots, K, \quad (7)$$

where  $b_k \in \{2, 4, 6, \dots\}$  and  $p_k \in [0, P_{\max}]$  with  $P_{\max}$  being the maximum allowed transmit power. Throughout this work, we assume  $P_{\max}$  is large.

For all linear receivers, the output SIR for user  $k$  can be written as

$$\gamma_k = (B/R_{s,k}) p_k \hat{h}_k \quad (8)$$

where  $B$  is the system bandwidth and  $\hat{h}_k$  is the effective channel gain which is independent of the transmit power and rate of user  $k$ . Based on (6) and (8), and by dropping the subscript  $k$  for convenience, the maximization in (7) can be written as

$$\max_{b, \gamma} B \hat{h} b \frac{f_b(\gamma)}{\gamma}. \quad (9)$$

It is important to observe that, for a given  $b$ , specifying the operating SIR completely specifies the utility function. Let us for now fix the symbol rate  $R_s$  and the constellation size. Taking the derivative of (9) with respect to  $\gamma$  and equating it to zero, we conclude that the utility of a user is maximized when its output SIR is equal to  $\gamma_b^*$  which is the unique (positive) solution of

$$f_b(\gamma) = \gamma f'_b(\gamma). \quad (10)$$

The maximum utility is hence given by

$$u_b^* = B \hat{h} b \frac{f_b(\gamma_b^*)}{\gamma_b^*} \quad (11)$$

We can compute  $\gamma_b^*$  numerically for different values of  $b$ . Table I summarizes the results. It is observed from Table I that the user’s utility is maximized when  $b = 2$  (i.e., QPSK modulation). This is because, as  $b$  increases, the linear increase in the throughput is dominated by the exponential increase in the required transmit power (which results from the exponential increase in  $\gamma_b^*$ ). As a result, it is best for a user to use QPSK modulation.<sup>2</sup> Fig. 1 shows the normalized user utility (i.e.,  $\frac{u_b}{B \hat{h}}$ ) as a function of SIR for different choices of  $b$ .

So far, we have shown that at Nash equilibrium (if it exists), QPSK modulation must be used by each user. The existence of the Nash equilibrium for the proposed game can be shown via the quasiconcavity of each user’s utility function in its own power. Furthermore, it can be shown that the equilibrium is unique (see [10] for more details).

<sup>2</sup>BPSK and QPSK are equivalent in terms of energy efficiency, but QPSK has a higher spectral efficiency.

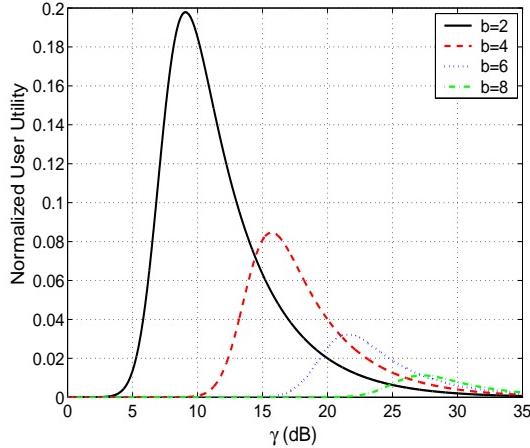


Fig. 1. Normalized user utility as a function of SIR for different constellation sizes.

#### IV. DELAY-CONSTRAINED POWER CONTROL GAME WITH M-QAM MODULATION

We now extend the analysis of Section III to the case in which the users have delay QoS requirements. Our goal in this part is to study the effects of constellation size on energy efficiency and delay. The delay QoS constraint considered here is in terms of average delay.<sup>3</sup>

Let us assume that the incoming traffic for user  $k$  has a Poisson distribution with parameter  $\lambda_k$  which represents the average packet arrival rate with each packet consisting of  $L$  bits. The user transmits the arriving packets at a rate  $R_k = b_k R_{s,k}$  (bps) and with a transmit power equal to  $p_k$  Watts. We assume an automatic-repeat-request (ARQ) mechanism in which the user keeps retransmitting a packet until the packet is received at the access point without any errors. Also, the incoming packets are assumed to be stored in a queue and transmitted in a first-in-first-out (FIFO) fashion. The packet success probability (per transmission) as before is represented by the efficiency function  $f_b(\gamma)$ .

Focusing on a specific user and dropping the subscript  $k$ , we can represent the combination of the user's queue and wireless link as an M/G/1 where the service rate,  $\mu$ , is given by

$$\mu = \frac{f_b(\gamma)}{\tau} = R_s \frac{b f_b(\gamma)}{L}. \quad (12)$$

Here,  $\tau$  represents the packet transmission time (i.e.,  $\tau = \frac{L}{b R_s}$ ). Now, let  $W$  be a random variable representing the total packet delay (including transmission and queuing delays) for the user. It can be shown that the average packet delay is given by (see [8] for details)

$$\bar{W} = \tau \left( \frac{1 - \frac{\lambda \tau}{2}}{f_b(\gamma) - \lambda \tau} \right) \quad \text{with } f_b(\gamma) > \lambda \tau. \quad (13)$$

The delay QoS constraint for a user is specified by an upper bound on the average packet delay, i.e., we require

$$\bar{W} \leq D. \quad (14)$$

<sup>3</sup>Note that an average-delay constraint may not be sufficient for applications with hard delay requirements (see [11]).

This delay constraint can equivalently be expressed as

$$\gamma \geq \hat{\gamma}_b$$

where

$$\hat{\gamma}_b = f_b^{-1}(\eta_b). \quad (15)$$

with

$$\eta_b = \frac{L\lambda}{bR_s} + \frac{L}{bR_s D} - \frac{L^2\lambda}{2b^2R_s^2D}. \quad (16)$$

This means that the delay constraint in (14) translates into a lower bound on the output SIR.

We propose a game in which each user chooses its transmit power and symbol rate as well as its constellation size in order to maximize its own utility while satisfying its delay requirement. Fixing the other users' transmit powers and rates, the best-response strategy for the user of interest is given by the solution of the following constrained maximization:

$$\max_{p, R_s, b} u \quad \text{s.t.} \quad \bar{W} \leq D, \quad (17)$$

or equivalently

$$\max_{\gamma, R_s, b} b \frac{f_b(\gamma)}{\gamma} \quad \text{s.t.} \quad \gamma \geq \hat{\gamma}_b \quad \text{and} \quad 0 \leq \eta_b < 1. \quad (18)$$

Let us define  $\Omega_b^* = \left(\frac{L}{D}\right)^{\frac{1+D\lambda+\sqrt{1+D^2\lambda^2+2(1-f_b^*)D\lambda}}{2f_b^*}}$ , where  $f_b^* = f_b(\gamma_b^*)$ .

*Proposition 1:* For given values of  $\lambda$  and  $D$ , the best-response strategy for a user (i.e., the solution of (17)) is any combination of  $p$  and  $R_s$  such that

$$\min \left\{ \Omega_b^*/\tilde{b}, B \right\} \leq R_s \leq B \quad (19)$$

$$\text{and} \quad \gamma = \begin{cases} \gamma_b^*, & \text{if } \Omega_b^*/\tilde{b} \leq B; \\ \hat{\gamma}_b, & \text{if } \Omega_b^*/\tilde{b} > B, \end{cases} \quad (20)$$

where  $\tilde{b}$  is the lowest constellation size for which  $\lambda$  and  $D$  are feasible,  $\gamma_b^*$  is the solution of (10), and  $\hat{\gamma}_b$  is given by (15).

*Proof:* The proof is omitted due to space limitation (see [10]). ■

Proposition 1 implies that, in terms of energy efficiency, choosing the lowest-order modulation (i.e., QPSK) is the best strategy unless the user's delay constraint is too tight. In other words, the user would jump to a higher-order modulation only when it is transmitting at the highest symbol rate (i.e.,  $R_s = B$ ) and still cannot meet the delay requirement. Also, the proposition suggests that if  $\Omega_b^*/\tilde{b} < B$ , the user has infinitely many best-response strategies.

It can be shown that for a matched filter, the utility of user  $k$  at Nash equilibrium is given by

$$u_k = \frac{B f(\gamma_k) h_k}{\sigma^2 \gamma_k} \left( 1 - \frac{\sum_{j \neq k} \Phi_j}{1 - \Phi_k} \right), \quad (21)$$

where  $\Phi_k = \left( 1 + \frac{B}{R_{s,k} \gamma_k} \right)^{-1}$  for  $k = 1, \dots, K$ .<sup>4</sup> This implies that while our proposed game could potentially have infinitely many Nash equilibria, the Nash equilibrium with the smallest  $R_{s,k}$ 's achieves the largest utility. This means the Nash equilibrium with  $R_{s,k} = \min\{\Omega_b^*/\tilde{b}_k, B\}$  for  $k = 1, \dots, K$  is the *Pareto-dominant* Nash equilibrium.

<sup>4</sup> $\Phi_k$  here is a generalized version of the "size" of user  $k$  as defined in [8].

## V. POWER CONTROL GAMES WITH TRELLIS-CODED M-QAM MODULATION

In this section, we extend our analysis to trellis-coded modulation (TCM). Let  $G$  represent the effective *coding gain* achieved by TCM as compared to the equivalent uncoded system (see [12]). In general, the coding gain is a function of both the operating SIR and the modulation level. Hence, the efficiency function for TCM is given by

$$f_b^{(c)}(\gamma) \simeq \left(1 - \alpha_b Q\left(\sqrt{\beta_b \gamma G_b(\gamma)}\right)\right)^{\frac{2L}{b}} - 2^{-L}, \quad (22)$$

where  $b$  is the number of *information* bits per symbol. One can follow the same analysis for the coded system as the one presented for the uncoded system by replacing  $f_b(\gamma)$  with  $f_b^{(c)}(\gamma)$  given in (22). Due to space limitation, we omit the analysis (see [10] for more details). We will show in Section VI that as expected, for the same spectral efficiency, the energy efficiency is higher when TCM is used.

## VI. NUMERICAL RESULTS

In this section, we quantify the effect of constellation size on energy efficiency of a user with a delay QoS constraint. The source rate (in bps) for the user is assumed to be equal to  $0.1B$  where  $B$  is the system bandwidth. We further assume that a user chooses its constellation size, symbol rate, and transmit power according to its best-response strategy corresponding to the Pareto-dominant Nash equilibrium (see Section IV). For the coded system, we assume an 8-state convolutional encoder with rate 2/3. The code rate for QPSK is chosen to be 1/2. Fig. 2 shows the “optimum” constellation size, transmit power, throughput, and user’s utility as a function of the delay constraint for both uncoded and coded systems.<sup>5</sup> For all four plots, the packet delay is normalized by the inverse of the system bandwidth. The throughput is obtained by multiplying the symbol rate by the number of (information) bits per symbol, and is normalized by the system bandwidth. The transmit power and user’s utility are also normalized by  $\hat{h}$  and  $B\hat{h}$ , respectively. Let us for now focus on the uncoded system. When the delay constraint is large, QPSK is able to accommodate the delay requirement and hence is chosen by the user. As the delay constraint becomes tighter, the user increases its symbol rate and also raises the transmit power to keep the output SIR at  $\gamma_b^* = 9.1\text{dB}$ . Eventually, a point is reached where the spectral efficiency of QPSK is not enough to accommodate the delay constraint. In this case, the user jumps to a higher-order modulation (i.e., 16-QAM) and the process repeats itself. The trends are similar for the coded system except that, due to coding gain, the required transmit power is smaller for the coded system. This results in an increase in the user’s utility (energy efficiency).

## VII. CONCLUSIONS

We have studied the effect of modulation order on energy efficiency of wireless networks using a game-theoretic framework. Focusing on M-QAM modulation, we have proposed a

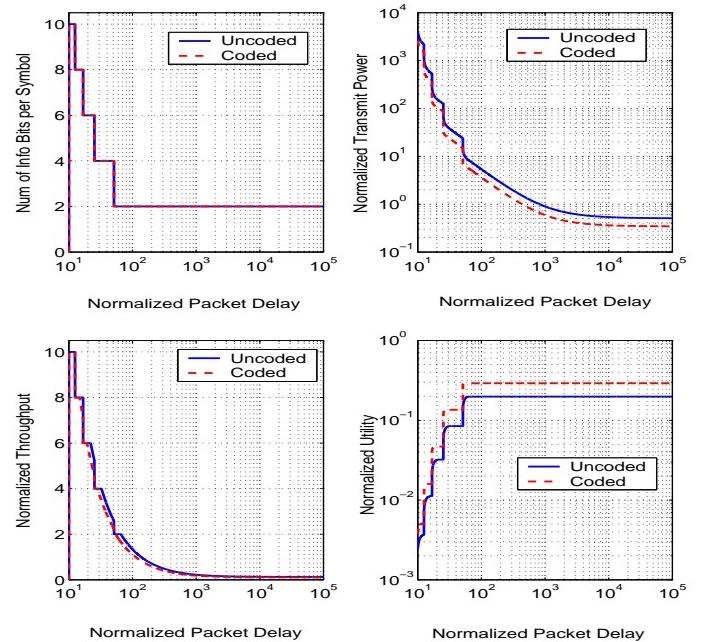


Fig. 2. Optimum modulation level, transmit power, throughput, and utility as a function of (normalized) packet delay.

non-cooperative game in which each user chooses its strategy in order to maximize its energy efficiency while satisfying its delay QoS constraint. The actions open to the users are the choice of the transmit power, transmit symbol rate and constellation size. The best-response strategies and the Nash equilibrium solution for the proposed game have been derived. Using our non-cooperative game-theoretic framework, the tradeoffs among energy efficiency, delay, throughput and constellation size have also been studied and quantified. In addition, we have included the effects of TCM and have shown that, as expected, coding increases energy efficiency.

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<sup>5</sup>Optimum here refers to the best-response strategy (i.e., the most energy-efficient solution).